Astronomy & Astrophysics manuscript no.
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Why the braking indices of young pulsars are less than 3?

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Abstract. In this letter we discuss two possible reasons which cause the observed braking indices n of young radio pulsars to be smaller than 3: (a) the evolving spin-down model of the magnetic field component $B_\perp$ increases with time; (b) the extrinsic braking torque model in which the tidal torques exerted on the pulsar by the fallback disk, and carries away the spin angular momentum from the pulsar. Based on some simple assumptions, we derive the expression of the braking indices, and calculate the spin-down evolutionary tracks of pulsars for different input parameters.

Key words. pulsars: general — Stars: magnetic fields — Stars: rotation

1. Introduction

Since the first radio pulsar was discovered by Hewish et al. (1968), continued radio observations at several wavelengths have not only increased the number of detected pulsars but also amassed a wealth of details about the temporal, spectral and polarization properties of the observed pulses. It was then quickly established that radio pulsars are rapidly rotating strongly magnetized neutron stars, powered by their rotational kinetic energy and losing energy by magnetic dipole radiation (Gold 1968). The spin angular velocity of pulsars decreases with time, and the spin-down may be written by the relation (Manchester & Taylor 1977),

$$I\dot{\Omega} = -K\Omega^n, \tag{1}$$

where Ω is the spin angular velocity, K is a positive parameter depending on the magnetic dipole moment, and n is the braking index. The braking index n can be a measured for young radio pulsars by differentiating equation (1),

$$n = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2}. \tag{2}$$

If the spin-down torque entirely results from magnetic dipole radiation with a constant magnetic field, the predicted braking index is n = 3. However, if the braking is due to a relativistic stellar wind, the braking index n = 1 (Michel 1969). The observed braking index n, which has been determined for several young radio pulsars, are all less than 3. For example, the braking index of the Crab pulsar PSR 0531+21 is 2.509 $\pm$ 0.001 (Lyne et al. 1993), n=2.140 $\pm$ 0.009 for PSR 0540-69 (Livingstone et al. 2005), n=2.837 $\pm$ 0.001 for PSR 1509-58 (Kaspi et al. 1994), n=2.91 $\pm$ 0.05 for PSR 1119-6127, n = 1.4 $\pm$ 0.2 for the Vela pulsar PSR 0833-45 (Lyne et al. 1996), and n = 2.65 $\pm$ 0.01 for PSR J1846-0258 which located at the center of supernova remnant Kes 75 (Livingstone et al. 2006).

In previous works, many authors proposed various models to explain the discrepancy between observed and theoretical predicted value of n. Peng et al. (1982) suggested that neutrino and photon radiation coming from superfluid neutrons may brake the pulsars. Blandford & Romani (1988) proposed that n < 3 is due to a multipole field and field evolution. Menou et al. (2001) and Alpar et al. (2001) presented a model that both magnetic dipole radiation and the propeller torque applied by the debris disk formed after supernova explosion can cause spin-down of pulsars. Xu & Qiao (2001) and Wu et al. (2003) pointed out that the combination of dipole radiation and the unipolar generator may cause n < 3. Allen & Horvath (1997) suggested that a variation of the torque function is important attribution for low braking index, and so on.

In this letter, we suggest that the braking index may be less than 3 due to increasing of magnetic field strength of pulsars or braking of fallback disks surrounding pulsars. In next section, we describe two simple models to explain why the observed braking index n < 3. In section 3 we present the calculated results for the braking model of the fallback disk. In section 4 we make a brief discussion and summary.

2. Model

Pulsar braking process is still poorly known even more than several decades after it was discovered. The best

radiation model is the magnetic dipole radiation, one which has been proven to be extremely successful in explaining the observed properties of pulsars (Pacini 1967; Gunn & Ostriker 1969). The rotating pulsars are braked by magnetic dipole radiation (Ostriker & Gunn 1969) or particle outflow from the poles of a dipole field (Goldreich & Jullian 1969) which has a constant field strength. However, many works propose that there are other energy loss mechanisms and braking torques besides magnetic dipole radiation,

$$I\dot{\Omega} = -K\Omega^3 + T(\Omega), \tag{3}$$

where $K = 2B^2R^6sin^2\theta/3c^3$; θ is the inclination of the magnetic axis with respect to the rotation axis; I, B, R are the momentum of inertia, the surface magnetic field strength, and the radius of the pulsar, respectively; c is the velocity of light, and $T(\Omega)$ is other braking torque.

We suggest that the observed braking index is less than 3 possibly due to the following two reasons: (a) $K = K(\Omega)$, $T(\Omega) = 0$, i.e. the other braking torque does not exist except for magnetic dipole radiation; (b) $K =$ constant, $T(\Omega) \neq 0$.

2.1. Case A: $K = K(\Omega), T(\Omega) = 0$

If the momentum of inertia of the pulsar is a constant, by differentiating equation (3), from equation (2) we can obtain the observed braking index

$$n = 3 + \frac{d\mathrm{lnK}}{d\mathrm{ln}\Omega}. \tag{4}$$

If the second term on the right-hand side of the above equation is negative, therefore $n < 3$ for young pulsars.

Assuming that the surface dipole magnetic field changes with angular velocity due to some unknown physical mechanisms, R, and the braking index n is constant during the pulsar spin-down, we can derive the evolution of magnetic field component $B_\perp = B\mathrm{sin}\theta$ in the direction perpendicular to the rotation axis with spin angular velocity of pulsar from equation (4)

$$B_\perp = C\Omega^{\frac{n-3}{2}}, \tag{5}$$

where C is the integrating constant determined by the parameters of the pulsar. Setting the initial spin angular velocity is Ω_0, and the magnetic field component is $B_{\perp,0}$ when pulsar was born, then

$$B_\perp = B_{\perp,0}\left(\frac{\Omega}{\Omega_0}\right)^{\frac{n-3}{2}}. \tag{6}$$

From equation (6), the magnetic field component $B_\perp$ is constant during spin down if n = 3. For young radio pulsars, $n < 3, \Omega < \Omega_0$ due to spin-down, we therefore can obtain $B_\perp > B_{\perp,0}$. Namely, the magnetic field vertical component $B_\perp$ of young pulsars would increase if the braking torque is only magnetic dipole radiation and the assumption mentioned in the above is true (Blandford & Romani 1988).

2.2. Case B: $K = \mathrm{constant}, \mathrm{T}(\Omega) \neq 0$

Actually, other braking torque possibly exist besides magnetic dipole radiation. Michel (1988) proposed that there may be disks surrounding radio pulsars, produced by supernova fallback. Menou et al. (2001) applied the fallback disk model and the propeller torque to explain the lower braking index of young radio pulsars. Following this idea we assume that there is a fallback disk residing outside the light cylinder of young pulsars, and tidal torques instead traditional propeller torques are put on the fallback disk, and carry away angular momentum from the pulsar inside it.

Same as standard thin disk, the viscous torque in the fallback disk is (Shakura & Sunyaev 1973)

$$T = 2\pi r\nu\Sigma r\frac{d\Omega_r}{dr}r, \tag{7}$$

where ν is the viscosity and Σ is the surface density, $\Omega_r = (GM/r^3)^{1/2}$ is the local rotational angular velocity in the fallback disk, and M is the mass of pulsar.

If the gravitational interaction of the pulsar with the fallback disk only occurs at the inner edge r_i of the fallback disk, angular momentum from pulsar feed into the fallback disk at a rate proportional to the surface density Σ_i at the inner region of the fallback disk. As a result of continuous input of mass at the inner edge of the fallback disk, the viscous torque exerted on the pulsar by the fallback disk can be shown to be (Taam & Spruit 2001; Spruit & Tamm 2001)

$$\dot{J}_\mathrm{d} = -\left(\frac{r_\mathrm{i}}{R}\right)^{1/2}\Omega R^2\dot{M}\left(\frac{t}{t_\mathrm{vi}}\right)^{1/2}, \tag{8}$$

where, t denotes the spin-down age of the pulsar, $\dot{M}$ is the mass inflow rate (assumed to be constant here), t_vi is the viscous timescale at the inner edge of disk.

Assuming that the fallback disk is hydrostatically supported and geometrically thin with a pressure scale height to radius ratio of $\beta = H_\mathrm{i}/r_\mathrm{i}$, we estimate the viscosity ν_i (Chen et al. 2006) at the inner edge of the fallback disk using standard α prescription presented by Shakura & Sunyaev (1973)

$$\nu_\mathrm{i} = \alpha_\mathrm{SS}\beta^2\Omega R^{3/2}r_\mathrm{i}^{1/2}, \tag{9}$$

where α_SS is the viscosity parameter. In the following calculations we set $\alpha_\mathrm{SS} = 0.001$.

By simple algebra, from the above equations we obtain the viscous timescale at the inner edge in the fallback disk,

$$t_\mathrm{vi} = \frac{4}{3\alpha_\mathrm{SS}\beta^2\Omega}\left(\frac{r_\mathrm{i}}{R}\right)^{3/2}. \tag{10}$$

The total braking torque consisting of magnetic dipole radiation and the fallback disk braking can be written as

$$I\dot{\Omega} = -K\Omega^3 + T_\mathrm{d}(\Omega), \tag{11}$$

with

$$T_\mathrm{d}(\Omega) = -\gamma^{-1}c^{-1/4}R^{9/4}\beta(3\alpha_\mathrm{SS}t/4)^{1/2}\dot{M}\Omega^{7/4}, \tag{12}$$

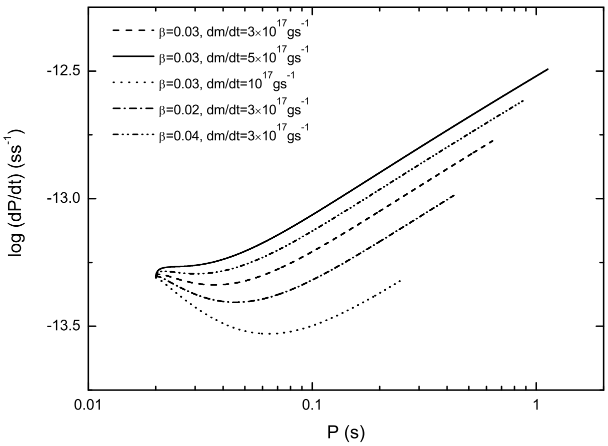

Fig. 1. The pulsar evolution in the $\dot{P}$ - P diagram for different input parameters β and the constant mass inflow rate $\dot{M}$ in the fallback disk.

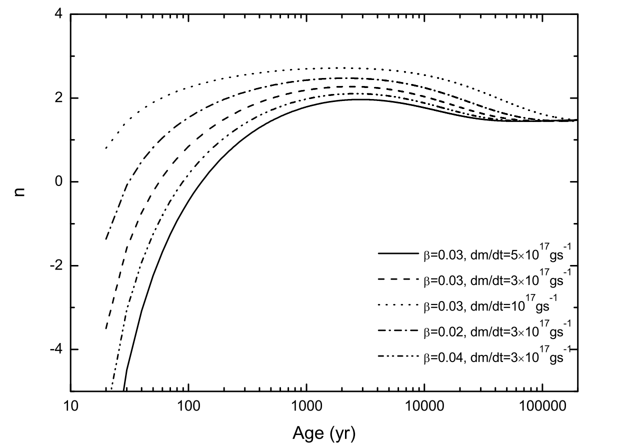

Fig. 2. Evolutionary tracks of braking index for different input parameters β and the constant mass inflow rate $\dot{M}$ in the fallback disk.

where $\gamma = (r_{\mathrm{i}}/R_{\mathrm{lc}})^{1/4}$, $R_{\mathrm{lc}} = c/\Omega$ is the light-cylinder radius of the pulsar. We assume the inner edge in fallback disk to be located at the light-cylinder radius (Menou et al. 2001), i.e. $\gamma = 1$.

Similar as in the above subsection, differentiating equation (11), from equation (2) we can get

$$n = 3 - (\frac{5}{4} + \frac{\tau}{t})\eta, \tag{13}$$

where $\tau = -\Omega/(2\dot{\Omega})$ is the canonical spin-down age of pulsars, $\eta = T_{\mathrm{d}}(\Omega)/(-K\Omega^3 + T_{\mathrm{d}}(\Omega))$ is the efficiency of the disk braking torque. The second term on the right-hand side of the above equation is positive, resulting in the braking index less than 3.

3. Calculated Results for Case B

We have adopted the semi-analytical method to calculate the evolution of $P, \dot{P}$, and n for radio pulsars with a fallback disk. We set some typical values for pulsars, $B_{\perp} = 10^{12}G$, $M = 1.4M_{\odot}$, $R = 10^6$cm, i.e. $K \simeq 2.5\times10^{28}\mathrm{g\cdot cm^2 s}$, $I \approx 10^{45}\mathrm{g\cdot cm^2}$; and $\beta = 0.02, 0.03, 0.04$ (Belle et al. 2004) in the calculations. Setting the initial spin period of the pulsar $P_{\mathrm{i}} = 0.02$s, we obtain the pulsar evolution in the $\dot{P}-P$ diagram and the evolutionary tracks of the braking index in Figs. 1 and 2 for different values of $\dot{M}$ and β, respectively. We can find that the braking index is all the way less than 3, and first increases then decreases during the spin-down from Fig. 2. The larger $\dot{M}$ and β, the larger $\dot{P}$ and the smaller n.

For the Crab pulsar, we can derive the current braking index since its age is a certain value in the case of a steady inflow in fallback disk . Set $t = 930$yr, the mass inflow rate $\dot{M} = 3\times10^{17}\mathrm{g\cdot s^{-1}}$, and $\beta = 0.03, 0.04, 0.05$, we derive the braking index are $n = 2.60, 2.49, 2.39$ from equation (13), which can account for the measured value within 5%.

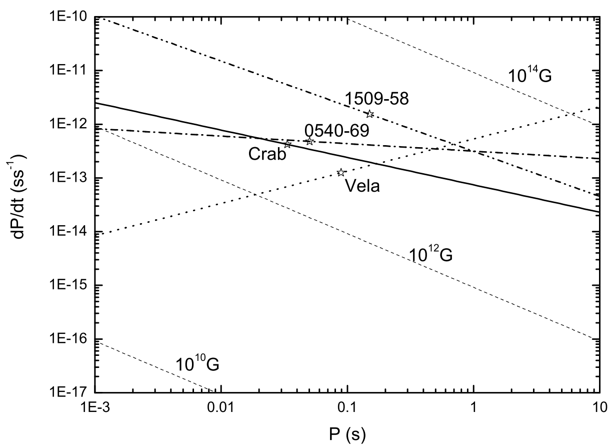

Fig. 3. Evolutionary tracks for four young radio pulsars, the solid, dot, dashed-dot, and dashed-dot dot lines correspond to the Crab pulsar, the Vela pulsar, PSR 0540-69, and PSR 1509-58, respectively. The four open stars denote the present location of four young pulsars.

4. Discussion and Summary

If the radiation process of pulsars is pure magnetic dipole radiation, and I, M, R are constant, the change of magnetic field strength is the possible reason of lower braking index. From $\dot{\Omega} \propto \Omega^{\mathrm{n}}$, we can derive the evolutionary tracks of four young pulsars in $\dot{P} - P$ diagram (see Fig. 3). Anomalous X-ray pulsars (AXPs) and soft Gamma-ray repeaters (SGRs) are a small class of pulsars with the relatively narrow period range of $5-12$s and a higher spin-down rates (Manchester 2001). The spin-down luminosity $\dot{E} = 4\pi^2 I\dot{P}/P^3$ of AXPs and SGRs is much less than their X-ray luminosity. Thompson & Duncan (1996) suggested that they are powered by decay of strong magnetic fields. It is clear that the magnetic field strength of Vela pulsar can exceed 10^{14} G when $P \sim 5-6$s from Fig. 3, suggesting that magnetars as AXPs and SGRs possibly evolve from normal radio pulsars as Vela (Lin & Zhang 2004).

However, the neutron star may lose spin angular momentum not only by magnetic dipole radiation but also by other braking mechanisms. Many authors proposed that young radio pulsars could be surrounded by a remnant disk generated by supernova fallback (Michel & Dessler 1981, 1983), which may contribute the braking torque causing spin-down of neutron stars. Assuming that a constant rate of material feeding into fallback disk, we derive the tidal torque exerted on the pulsar by the fallback disk. Our calculations show that the braking torque is significantly influenced by the input parameters $\dot{M}$ and β. The larger values of $\dot{M}$ or β can yield a larger braking torque and a smaller braking index n. Fig. 1 shows that large $\dot{M}$ or β can cause the generation of pulsars with high spin-down rate, like SGRs and AXPs. The evolution of the braking index shows that they are all less than 3 during spin-down (see Fig. 2).

If some of the so-called magnetar, i.e. SGRs and AXPs, evolve from normal young pulsars as described by case a, their surface magnetic field should be strong as $\gtrsim 10^{14}$ G, while in case b they may have usual magnetic field $\sim 10^{12}$G but with high spin-down rates. Measurement of cyclotron lines in magnetars may present constrains on the two kinds of models. In addition, the presence of fallback disks surrounding pulsars could be detected, since a relatively dense fallback disk can produce optical and infrared radiation. Optical emission is a common character of young pulsars. The Crab pulsar, the Vela pulsar, and PSR 0540-69, which possess low braking index, have been shown to possess a pulsating optical counterpart (Menou et al. 2001), implying the possible existence of the fallback disks. Searching for optical and infrared emission from the fallback disks surrounding young pulsars will be of great importance for their evolution.

Acknowledgements. We are grateful for an anonymous referee for helpful comments on this manuscript. This work was supported by the Natural Science Foundation of China (NSFC) under grant number 10573010.

References

Allen, M. P., & Horvath, J. E. 1997, ApJ, 488, 409
Alpar, M. A., Ankay, A.,& Yazgan, E. 2001, ApJ, 557, L61
Belle, K. E., Sanghi, N., Howell, S. B., Holberg, J. B., & Williams, P. T. 2004, AJ, 128, 448
Blandford, R. D., & Romani, R. W. 1988, MNRAS, 234, 57
Chen, W. C., Li, X. D., & Wang, Z. R. 2006, PASJ, 58, 153
Gold, T. 1968, Nature, 218, 731
Guliahorn, G. E., & Rankin, J, M. 1978, ApJ, 83, 10
Gunn, J. E., & Ostriker, J. P. 1969, Nature, 221, 454
Goldreich, P., & Jullian, W, H. 1969, ApJ, 157, 869
Hewish, A., Bell, S. J., Pilkington, J. D., Scott, P. F., & Collins, R. A. 1968, Nature, 217, 709
Kaspi, V. M., Manchester, R. N., Siegman, B., Johnston, S., & Lyne, A. G. 1994, ApJ, 422, L83
Lin, J. R., & Zhang, S. N. 2004, ApJ, 615, L133
Livingstone, M. A., Kaspi, V. M., & Gavriil, F. P. 2005, ApJ, 633, 1095
Livingstone, M. A., Kaspi, V. M., & Gotthelf, E. V. 2006, submitted to ApJ(astro-ph/0601530)
Lyne, A. G., Pritchard, R. S.,& Smith, F. G. 1993, MNRAS, 265, 1003
Lyne, A. G., Pritchard, R. S., Graham-Smith, F., & Camilo, F. 1996, Nature, 381, 497
Manchester, R. N. 2001, in AIP Conf. Proc. 565, 305
Manchester, R. N., & Peterson, B. A. 1989, ApJ, 342, L23
Manchester, R. N., & Taylor, J. H. 1977, pulsars, San Francisco, Freeman
Melatos, A. 1997, MNRAS, 288, 1049
Menou, K., Perna, R., & Hernquist, L. 2001, ApJ, 554, L63
Michel, F. C. 1969, ApJ, 158, 727
Michel, F. C., & Dessler, A. J. 1981, ApJ, 251, 654
Michel, F. C., & Dessler, A. J. 1983, Nature, 303, 48
Michel, F. C. 1988, Nature, 333, 644
Ostriker, J. P., & Gunn, J. E. 1969, ApJ, 157, 1395
Pacini, F. 1967, Nature, 216, 567
Peng, Q. H., Huang, K. L., & Huang, J. H. 1982, A&A, 107, 258
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Simon, J., & David, G. 1999, MNRAS, 306, L50
Spruit, H. C., & Taam, R. E. 2001, ApJ, 548, 900
Taam, R. E., & Spruit, H. C. 2001, ApJ, 561, 329
Thompson, C., & Duncan, R. C. 1996, ApJ, 473, 322
Wu, F., Xu, R. X., & Gil, J. 2003, A&A, 409, 641
Xu, R. X., & Qiao, G. J. 2001, ApJ, 561, L85